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The Slow Drift Oscillations of a Moored Object in Random Seas

Ву

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ABSTRACT

In this paper the phenomenon of the slowly varying drifting force on a moored object in a random sea will be explained and illustrated from the results of a number of model tests with a rectangular barge. These tests, conducted at the Netherlands Ship Model Basin, were an extension of an earlier executed program [].

It will be shown that, using the results of measured or calculated drifting forces on an object moored in regular waves, a prediction can be made of the drifting forces induced by wave trains consisting of regular wave groups.

Also for an irregular wave train the drifting force on the barge could be computed as a function of time, which made it possible to calculate the surge motion of the barge

References and illustrations at end of paper

The results of tests and calculations show a reasonable agreement.

INTRODUCTION

In the last few years the problems concerning the mooring of objects in random seas have gained much attention as a result of the necessity to load and discharge big tankers in open sea or due to the fact that the sea bottom had to be explored and exploited from vessels operating from the water surface.

A floating object moored in waves will generally be subjected to forces causing horizontal and vertical motions and moments causing angular motions about horizontal and vertical axes. In this paper the horizontal surge motion of a rectangular barge moored by means of linear springs in head waves will be dealt with. The surge motion can be split-up in a mean excursion, a slowly varying motion and a higher frequency oscillation around the slowly varying position.

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The period of the higher frequency oscillation is equal to that of the wave motion and since a considerable amount of literature is available concerning this part of the motion, it will not be treated in this paper. From the results of model tests in regular waves the mean drifting force on the barge could be determined as a function of the wave frequency. Using these data the long periodical surge motion of the barge was calculated for different stiffnesses of the mooring system for the condition that the barge was moored in a wave train which consisted of regular wave groups. The results of those calculations are compared with model test results. From these and earlier executed tests [1] it is clear that resonance phenomena may occur when the period with which the wave groups encounter the barge equals the natural period of the surge motion of the moored barge. Further it appears to be possible to calculate the drift force induced by regular wave groups when such a wave train is considered to consist of two regular waves with a small difference in frequency.

However, regular wave groups will seldom occur. Generally the wave height changes irregularly. To estimate the drift forces exerted on an object in irregular waves as a function of time there exists a method which produces reasonable results. This method will be dealt with. Starting from the so obtained drifting force, the surge motion of the object moored in this particular wave train can be calculated. This is illustrated by comparison of some calculated and measured surge motion records.

THE DRIFT FORCE IN REGULAR WAVES

The hydrodynamic forces on an object floating in regular waves may be resolved in an oscillatory part and in a constant part of which the latter is known as the steady drifting force. Maruo [2] shows for the two dimensional case of an infinitely long cylinder floating in regular waves with its axis perpendicular to the wave direction that the steady drifting force Fd per unit length satisfies:

 $F_d = \frac{1}{2} \rho g a^2$

in which:

- ρ = specific mass density of water

 - g = acceleration due to gravity a = amplitude of the wave reflected and scattered by the body in a direction opposite to the incident wave

Generally only a part of the incident wave will be reflected; the rest will be transmitted underneath the cylinder. Besides the reflected wave also waves are generated opposite to the incident wave due to the heave, sway and roll motions of the cylinder. All reflected and scattered waves have the same frequency which means that the sum of these components is again a regular wave with the same frequency and with an amplitude depending on the amplitudes of the reflected and scattered wave components and their mutual phase differences. The amplitudes of these components and the phase differences depend on the frequency of the incident wave, while the amplitudes can be assumed to be linearly proportional to the amplitude of the incident wave. This means that the steady drifting force F_d in regular waves per unit length of the cylinder can be written as:

$$F_{d} = \frac{1}{2} \rho g \left[R (\omega) \cdot \zeta_{a} \right]^{2}$$
(1)

in which:

R (ω). ζ	-	amplitude of reflected and scattered wave
, a		and scattered wave
R (ω)	=	reflection coefficient
ζa	=	amplitude of incident
- a		wave
ω	=	frequency of incident
		wave

This expression indicates that the drifting force is proportional to the square of the wave height.

To verify this and to determine the reflection coefficient R for a rectangular barge in head waves, tests were conducted in the Wave and Current Basin of the N.S.M.B. In this basin (length 60 m; width 40 m; maximum water depth 1.20 m) a model of a rectangular barge was moored to a fixed point by means of a bow hawser, which consisted of a linear spring with spring constant C.

To avoid the bow hawser becoming slack, a countermass was used as is indicated in Figure 1. In this Figure the test set-up and the particulars of the tested barge are given.

In the present paper all dimensions are given for the full size barge in sea water. The model scale is 1:80.

By measuring the force in the bow hawser and the surge motion of the barge in a number of regular head waves of different frequencies and by recording both signals on magnetic tape, the mean drifting force F could be determined with reasonable accuracy. From this a reflection coefficient R was determined using

$$R = \sqrt{\frac{F_d}{\frac{1}{2} \rho g \zeta_a^2 B}}$$
(2)

in which B = breadth of the barge.

For two different amplitudes of the stroke of the wave generator, which resulted in different wave heights at the same frequency, the so obtained reflection coefficients R have been plotted in Figure 2. From these results it can be concluded that the drifting force is indeed.approximately proportional to the square of the wave height.

The heave and pitch motions which also influence the magnitude of the drifting force were measured.

In Figure 3 and 4 the dimensionless amplitudes of the heave and the pitch motion have been plotted as a function of the frequency of the waves. The measured results are compared with the values obtained from calculations using a modified strip theory according to Flokstra [3]. Since the agreement between the measured and calculated heave and pitch motion is acceptable, an attempt was made to determine the mean drift force according to Havelock

4 using the calculated quantities and the following expression: 3

$$F_{d} = \frac{1}{2} \frac{\omega}{g} (Z_{a}^{2} \cdot Q_{zz} + \theta_{a}^{2} \cdot Q_{\theta\theta})$$

in which:

Z,		amplitude of heave motion
Θa		amplitude of pitch motion
Za Oa Qzz	=	damping coefficient for
2.2		heave motion
^{କ୍} ee	=	damping coefficient for pitch motion

With this formula Havelock approximates the mean increase of resistance in regular waves by integrating the wave pressure over the submerged part of the hull of the heaving and pitching ship. He assumes that the pressure will not be influenced by the presence of the ship. However, due to the flat bow and bottom of the tested barge wave reflection plays an important role, which means that his assumption is not valid and unacceptable for the tested barge.

To illustrate the influence of the wave reflection on the drifting force on the barge, the reflection coefficient R according to Haskind [5] for a captive vertical plate with a draft equal to the draft of the barge has been plotted in Figure 2. To indicate the possible influence of the bottom of the barge on the wave reflection, the wave reflection coefficient R according to Stoker [6] and Mei [7] for a captive barge of zero draft (infinite breadth) and with a length equal to the length of the tested barge has been given in Figure 2.

For an exact theoretical treatment of the drifting force problem for a floating body reference may be made to [8].

THE DRIFT FORCE IN REGULAR WAVE GROUPS

From the expression for the mean drift force in regular waves it will be clear that for the case that the wave height varies slowly in time with a certain period, the drift force on an object will also vary slowly with the same period. To illustrate this we consider a wave train ζ which consists of two regular components which have a small difference in frequency.

$$\zeta = \zeta_1 \cos (kx - \omega t + \varepsilon_1) +$$

+
$$\zeta_{2} \cos ((\mathbf{k} + \Delta \mathbf{k}) \mathbf{x} - (\omega + \Delta \omega) \mathbf{t} + \varepsilon_{2}$$

This can be written as:

$$\zeta = a \cos (kx - \omega t - \theta)$$

in which a = envelope of the wave elevation or slowly varying wave amplitude The drift force is proportional to the square of the wave amplitude, which satisfies:

$$a^{2} = \zeta_{1}^{2} + \zeta_{2}^{2} + 2\zeta_{1}\zeta_{2} \cos (\Delta kx - \Delta wt + \varepsilon_{1} - \varepsilon_{2})$$
(3)

The value of a^2 can also be determined by taking the square of the wave motion ζ , for

$$\zeta^{2} = \frac{1}{2} (\zeta_{1}^{2} + \zeta_{2}^{2}) + \zeta_{1} \zeta_{2} \cos (\Delta kx - (4))$$

$$-\Delta \omega t + \varepsilon_{1} - \varepsilon_{2}) + \text{high frequency}$$
components

According to (2) the drift force on the barge moored in such a wave train can be calculated from:

$$F_{d} = \frac{1}{2} \rho g a^{2} R^{2} (\omega) B \qquad (5)$$

It will be clear that the drift force consists of a constant part and a slowly oscillating part with frequency $\Delta \omega$.

The constant part $\overline{F_d}$ of the drifting force satisfies:

$$\overline{F}_{d} = \frac{1}{2} \rho g (\zeta_{1}^{2} + \zeta_{2}^{2}) R^{2} (\omega) B$$
(6)

while the amplitude F of the slowly oscillating part can be written as:

$$F_{da} = \rho g \zeta_1 \zeta_2 R^2 (\omega) B \qquad (7)$$

To verify these expressions a number of tests have been conducted with the rectangular barge moored by various linear springs in a number of different spectra representing wave trains consisting of regular wave groups. All wave groups had a period of about 111 sec.

For 3 wave spectra the particulars are given in Table I. The wave height was measured by a resistance type wave probe and was recorded on magnetic tape. With the aid of a digital computer the square of the wave motion was determined as a function of time. For the so obtained signal the mean value ζ^2 and the maximum harmonic component (Γ_{gr}) with a period in the vicinity of the group period Γ_{gr} were computed. As an example the recorded wave train E 5-11 was copied from the paper chart and is given in Figure 5. In this Figure the computed envelope is also shown; this was calculated from:

$$a = \sqrt{2 \overline{\zeta}^2 + 2 \zeta_a^2(T_{gr})} \cos (\Delta \omega t)$$
in which $\Delta \omega = \frac{2\pi}{\pi}$
(8)

The computed wave envelope corresponds well with the actual envelope.

~gr

In Table I numerical values of the mean value ζ^2 and the harmonic component ζ^2 (T_p) are given for three different spectra. These spectra are plotted in Figure 6.

The mean drift force $\overline{F_2}$ and the amplitude of the slowly oscillating force component $\overline{F_2}$ which will be exerted on the barge when it is moored in the regular wave groups can now be approximated according to (6) and (7), which leads to:

$$\overline{F_{3}} = \rho g \overline{\zeta}^{2} R^{2} (\omega) B \qquad (9)$$

$$F_{a3} = \rho g \zeta_a^2 (T_{gr}) R^2 (\omega) B \quad (10)$$

R is determined from Figure 2 for

$$\omega = \frac{2\pi}{\overline{m}}$$

\overline{T} = mean period of waves

The computed forces $\overline{F_{x}}$ and $\overline{F_{x}}$ were compared with the results of the tests with the barge moored by means of different springs in the particular wave train. In each wave train six different mooring springs were investigated. From extinction tests in still water the damping coefficient b and the natural period of the surge motion were determined. The springs were selected in such a way that they induced natural surge periods T_{n} from 65 to 160 sec.

During each test the surge motion x and the force in the bow hawser F_{bh} were measured. From the records the mean values $\overline{x_2}$ and $\overline{F_{bh}}$ and the maximum harmonic amplitudes x_{a2} (for $T \approx T_{gr}$) and Fbha were determined.

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Finally

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from:

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$$\overline{a^{2}(t)} = \sum_{n=1}^{\infty} \zeta_{n}^{2} = \sqrt{2} S_{\zeta}(\omega) d\omega$$
(13)

As the drifting force is proportional to the square of the wave height or in other words to the square of the wave envelope, the mean drifting force F_1 may be calculated from:

$$\overline{F_1} = \rho g.B \int S_{\zeta} (\omega).R^2(\omega) d\omega \qquad (14)$$

(see also equation (2))

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This expression was used to calculate the mean drift force exerted by the regular wave groups on the barge In Table I the so calculated mean drift force $\overline{F_1}$ is compared with the measured value $\overline{F_2}$. The agreement is good, which was already shown by Gerritsma [9] and others.

The results obtained with the above, given formula were also compared with the results of tests with the barge in a fully irregular sea with spectrum code P_2 (see Figure 6). Six different springs were tested. The results are given in Table II.

THE SLOWLY VARYING FORCE IN IRREGULAR WAVES

As has already been shown the square of the wave motion provides information about the square of the slowly varying wave envelope of an irregular wave train and so also about the drifting force. In principle a spectral analysis may be made of the square of the wave envelope reduced by the mean value.

In other words the spectral density of the square of the wave motion provides information about the mean period and the magnitude of the slowly varying drift force. However, in practice it is very

However, in practice it is very difficult to obtain an accurate wave envelope spectrum due to the long wave record which is required. Assuming that 200-250 oscillations are required for an accurate spectral analysis and that the mean period of the wave envelope record amounts to about 100 sec., the total time that the wave height has to be recorded amounts to 5.5 to 7 hours.

Therefore it was decided to use a method as described by Hsu in [10] . This method is based on the assumption that an irregular wave train consists of a sequence of single waves of which the height is characterized by the height of the wave crest or the depth of the wave trough, while the period is determined by the two adjacent zeró crossings (see Figure 7). Each of the so obtained single waves (one for every top or trough) is considered to be one of a regular wave train, which exerts a drifting force on the barge:

$$F_n = \frac{1}{2} \rho g \zeta_n^2 .R^2(T_n).B$$
 (15)

in which:

- ζ_n = height of wave crest or depth of trough
- In = twice the time period elapsed between two adjacent zero crossings.

The so obtained irregular slowly varying drifting force F(t) (from now on indicated by F_{μ}) on the barge in a certain wave train will induce surge motions x, which can be calculated by solving the well known differential equation:

$$\mathbf{m}\mathbf{\ddot{x}} + \mathbf{b}\mathbf{\dot{x}} + \mathbf{c}\mathbf{x} = \mathbf{F}(\mathbf{t}) = \mathbf{F}_{1}$$

For the investigated and tested case of linear mooring springs, using constant damping and mass coefficients, the surge motion can be determined for t sufficiently large from:

 $x(t) = \int h(\tau) \cdot F(t-\tau) d\tau = x_{4} \quad (16)$ in which $h(\tau) = \text{impuls response}$ function $h(\tau) = c_{1} \cdot e^{c_{2} t} \sin c_{3} t \quad (17)$ $c_{1} = \frac{2}{\sqrt{4 \text{ mc} - b^{2}}}$ $c_{2} = \frac{-b}{2m}$ $c_{3} = \frac{1}{2m} \quad \sqrt{4 \text{ mc} - b^{2}}$

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Hsu [10] has shown that this produces reliable results. I be illustrated by comparing	s method
produces reliable results.	This will
be illustrated by comparing	some test
results with the calculated	values
x_4 and F_4 .	

TESTS WITH BARGE IN REGULAR WAVE GROUPS

From the record of the wave motion of the regular wave group, the drifting force signal F4 was calculated by computer. The result could be resulved in a constant part $\overline{F_{\mu}}$ and an oscillatory component . The period of the oscillatory part was found to be equal to that of the wave groups (111 sec.). As an example the values of \overline{F} and $F_{a\mu}$ are given in Table I for three spectra. The values show good agreement with the F agreement with the measured values \overline{F}_2 and \overline{F}_2 , except for spectrum E^25-9 for which the calculated value of the force amplitude is about 30% higher than the measured value.

In Figure 5 the calculated values of the surge amplitude x have been plotted for the six different springs for spectrum E 5-11. These values were obatined from equation (12) using F_{a4} instead of F_{a3} .

2. TESTS WITH THE BARGE IN IRREGULAR WAVES

In the irregular wave train, which is described by the spectrum P₃, (see Figure 6) six tests were conducted with the barge moored with different linear springs. During these tests, which lasted about 35 minutes (full scale time), the surge motion and the force in the bow hawser were measured and recorded on magnetic tape. These signals were filtered in order to smooth out the (small) contribution due to the higher frequency surge motion with periods in the vicinity of the wave motion. The so obtained signals were analysed. In Table III the following quantities are given for the surge motion:

- mean value
- root mean square value
- maximum forward motion maximum backward motion

The measured quantities are compared with the results of the calculations according to the equations (16) and (17). The agreement appears to be good. In Table II the mean drifting force F_4 obtained from $F_4 = c.x_4$ is given for the different springs. As could be expected the value of the calculated mean force corresponds reasonably with the measured values and the values calculated from the wave spectrum.

CONCLUSIONS

It appears to be possible to determine the slowly oscillating character of the surge motion of a moored vessel in a given irregular wave train when the reflection coefficients R are known as a function of the wave frequency ω . Probably this holds also for other wave directions, providing that the wave reflection coefficients are known for these directions.

It is clearly shown that reasonance may occur when wave groups are present which encounter the ship with a period in the vicinity of the natural period of the mooring system. Probably 4 or 5 of such wave groups may well induce severe horizontal motions.

Therefore more information will be required concerning the occurrence of wave groups on the locations where big vessels are to be moored.

When short-time wave records for a particular location are available and the reflection coefficient R is measured, in a large model basin, for the ship under consideration, a reasonably accurate prediction of the behaviour of the vessel may be made using the method of calculation outlined in this paper. Since the equation of motion is solved numerically the method may also be applied when non-linear bow hawsers or mooring systems are used.

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NOMENCLATURE	REF
Indices a = amplitude bh = bow hawser	[1]
<pre>1 = calculated from spectral density acc. eq. (14) 2 = measured or obtained from model tests 3 = computed from regular components acc. eq. (9), (10), (12) 4 = computed from wave record acc. eq. (15)</pre>	[2]
B = breadth of barge F = force amplitude	3
<pre>F^a = mean force L = length of barge Q_z = damping coefficient for heave motion of barge Q₀₀ = damping coefficient for pitch motion of barge R = reflection coefficient Sg = spectral density T = draft of barge</pre>	[4]
$\overline{\overline{T}}$ = mean period of wave spectrum T_{gr} = period of wave groups T_{n}^{gr} = natural period of surge motion	[5]
<pre>a = wave envelope b = damping coefficient by slowly surge motion</pre>	*
c = spring constant g = acceleration due to gravity h = water depth k = wave number $\frac{2\pi}{2}$	[6]
<pre>m = total mass of barge and mooring system t = time x = horizontal alongships coordinate,</pre>	[7]
z = amplitude of heave motion $\varepsilon = phase angle$	[8]
$\zeta = vertical motion of water surface \zeta = vertical motion of water surface \zeta = vertical motion of vert and vert and$	9
$ \begin{aligned} \boldsymbol{\zeta}_a^2 & (\mathbf{T}_{gr}) &= \text{maximum harmonic component} \\ & \boldsymbol{\sigma}_f^2 & \text{with period } \mathbf{T}_g \\ \boldsymbol{\theta} &= \text{amplitude of pitch motion} \\ \boldsymbol{\theta} &= \text{wave length} \\ \boldsymbol{\lambda} &= \text{dimensionless frequency} &= \mathbf{T}_n / \mathbf{T}_g \\ \boldsymbol{\rho} &= \text{specific mass density of water} \\ \boldsymbol{\sigma} &= \text{root mean square value} \\ \boldsymbol{\omega} &= \text{frequency} \end{aligned} $	[10]

REFERENCES

- Hermans, A.J. and Remery, G.F.M.
 "Resonance of moored objects in wave trains".
 Proc. of the 12th Conference on Coastal Engineering
- 2] Maruo, H. "The drift of a body floating on waves". Journal of Ship Research. Vol. 4; December '60.
- Flokstra, C.
 "Ship motions in regular waves".
 Yet to be published by the Netherlands Ship Model Basin 1971.
- 4] Havelock, T.H. "The drifting force on a ship among waves". Philosophical Magazine, Ser.7, Vol. XXXIII.p.467, June 1942.
- [5] Haskind, M.D.
 "The pressure of waves on a barrier".
 see "Encyclopedia of Physics".
 Springer Verlag, Berlin '60.
 "Surface waves" of Wehausen and Laitone. sect. 17.
- 6] Stoker, J.J. "Water Waves". New York : Interscience 1957.
- 7] Mei, C.C. and Black, J.L. "Scattering of surface waves" J. Fluid Mech. (1969) Vol. 38 Part 3. p.p. 433-455.
- 8] 60th Anniversary Series. Soc. of Naval Architects of Japan. Vol. 8. Chapter 5.
- 9 Gerritsma, J., Bosch, v.d. J.J. and Beukelman, N. "Propulsion in regular and irregular waves". International Shipbuilding Progress, 1961.

[10] Hsu, F.H. and Blenkarn, K.A. "Analysis of Peak Mooring Force caused by Slow Vessel Drift oscillation in random seas". 0.T.C. 1970, Paper no. 1159.

	Dimension	Spectrum			
	Dimension	E5-7	E5-9	E5-11	
Ğ ₩1/3	m	5.67	5.97	5.01	
T T	sec.	7.37	9.36	12.16	
<u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u><u></u></u>	m2	2.04	2.28	1.58	
$\zeta_a^2 (T_{gr.})$	m ²	1.11	2.27	1.58	
^T gr.	sec.	111.70	112.30	111.50	
F ₁	ton	80.00	96.00	37.00	
F ₂	ton	81.00	96.00	42.00	
F ₃	ton	94.00	104.00	38.00	
F ₄	ton	90.00	93.00	38,00	
F _{a2}	ton	49.00	63.00	40.00	
Fa3	ton	51.00	104.00	37.00	
Fa4	ton	50.00	92.00	38.00	

TABLE 2 - MEAN DRIFTING FORCE IN SPECTRUM P3

- -

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	Spring constant ,C in ton.m ⁻¹	Measured mean <u>f</u> orce F2 in ton	Calculated mean force $F_4 = x_4$. C in ton
ĺ	136	28	28
	68	27	29
	49	22	30
	40	30	30
	29	26	30
	20	28	30

Calculated $\overline{F_1}$ = 28 ton

TABLE 3 - COMPARISON OF CALCULATED AND MEASURED SURGE MOTION IN IRREGULAR WAVES WAVE SPECTRUM P3, VALUES OBTAINED FROM A 35-MINUTE RECORD

Spring		Mean	Root mean	Maximum	Maximum
constant		value	square value	backward	forward
C		X	^G	motion	motion
in ton.m ⁻¹		in m	in m	in m	in m
136	Measured	0.21	0.41	1.62	1.03
	Calculated	0.21	0.48	1.77	1.27
68	Measured	0.37	0.44	2.31	1.00
	Calculated	0.43	0.52	2.44	1.15
49	Measured	0.45	0.65	3.07	1.39
	Calculated	0.61	0.65	2.98	1.05
40	Measured	0.74	0.67	3.62	1.07
	Calculated	0.75	0.77	3.46	1.04
29	Measured	0.90	0.82	3.90	1.33
	Calculated	1.04	1.07	4.52	1.27
20	Measured	1.40	1.09	5.06	1.45
	Calculated	1.50	1.26	5.23	1.28

