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RESPONSE OF A RANDOMLY INHOMOGENEOUS LAYER OVERLYING A HOMOGENEOUS HALF-SPACE TO SURFACE HARMONIC EXCITATION

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SUMMARY

The paper is concerned with the study of the effect of the randomness of material parameters on mean wave propagation in a semi-infinite viscoelastic medium. The medium considered in this paper is a configuration of a randomly non-homogeneous layer overlying a homogeneous half-space and is loaded harmonically on the top surface. The method used is that of Karal and Keller and is based on the idea of fundamental matrix and Bourret approximation. The integro-differential equation obtained in this paper is solved by the Laplace transform method. By using boundary and continuity conditions the mean wave solution is obtained. Numerical results show that the correlation functions, which introduce long-range interactions, can be the source of the wave amplification.

1. INTRODUCTION

The properties of waves propagating in a randomly non-homogeneous viscoelastic medium are governed by differential equations whose coefficients are random functions of space variables.[1,2] In this paper we are dealing with a displacement field related to a stress field by means of the equation of motion, the constitutive equation and the boundary conditions.

In engineering practice, in order to describe the macroscopic behaviour of the random medium, we do not need the displacement field itself, which exhibits random fluctuations, but rather some of its statistics. In the simplest case we may ask for the mean field. Although we are mostly interested in three-dimensional boundary-value problems, it is sometimes desirable to reduce the number of dependent and independent variables of the deformation field by making certain simplifying assumptions. The simplest physical regime is that in which stresses depend upon a single Cartesian coordinate, say z (see Figure 1).

The physical problem we wish to solve reduces to determining the motion of the interior of a randomly non-homogeneous viscoelastic semi-infinite medium when a harmonic pressure pulse is applied to the top surface. This problem arises naturally in the study of dynamically loaded structures.[3] Although in recent years the research in the area of dynamic soil–structure interaction was remarkably successful in explaining the effects of various factors on the behaviour of a deformable body,[4,5] the steadily increasing precision of measurements and observations brought forth the need for modification of certain points in the existing theories.[6,7] The unquestionable fact is that a real soil is a kind of medium whose properties are very complex and are not known precisely. Many investigators have been aware of the need to account for uncertainty in the parameters appearing in the equations of motion in a quantitative manner.

The basic idea in the derivation here is that of Karal and Keller.[8] The resulting mean wave integro-differential equation for the dynamics of the medium obtained in this paper is similar to those in non-local

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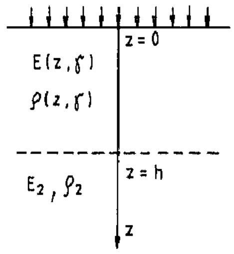

Figure 1. Geometry of the problem

continuum theories.[7] The correlation functions of the random medium which introduce long-range interactions can be the sources of the wave amplification.

2. BASIC EQUATIONS AND ASSUMPTIONS

Consider wave propagation in a randomly non-homogeneous viscoelastic semi-infinite medium. The simplest equation which describes it is

$$\frac{\partial \tau}{\partial z}=\rho(z,\gamma)\frac{\partial^2 w}{\partial t^2} \tag{1}$$

with the constitutive equation

$$\tau=E(z,\gamma)\frac{\partial w}{\partial z}+\xi\frac{\partial^2 w}{\partial t\,\partial z} \tag{2}$$

where w denotes the displacement, τ the stress tensor, t time, γ the probability variable and $z\geqslant 0$ is the space variable. The damping coefficient ξ is assumed to be constant and deterministic. The density of the medium, ρ, and the modulus of elasticity, E, can be expressed as the sum of their average value and fluctuation

$$\rho(z,\gamma)=\rho_{01}(1+\varepsilon_1(z,\gamma)) \tag{3a}$$

$$E(z,\gamma)=E_{01}(1+\varepsilon_2(z,\gamma)) \tag{3b}$$

where $\langle\rho\rangle=\rho_{01}$, $\langle E\rangle=E_{01}$, $\langle\varepsilon_1\rangle=\langle\varepsilon_2\rangle=0$.

The angular brackets $\langle.\rangle$ are used for denoting the average value (mean value). It is to be noted that, assuming Poisson's ratio $\nu=0{\cdot}25$, we have $E_{01}=\mu_0$ for shear waves and $E_{01}=3\mu_0$ for dilatational waves, where μ_0 denotes the average value of the shear modulus of the medium.[5]

If the excitation on the top of the medium is

$$\tau(z,t)|_{z=0}=\tau_0\exp(\mathrm{i}\omega t) \tag{4}$$

then in the steady state

$$w(z,t)=w(z)\exp(\mathrm{i}\omega t) \tag{5}$$

where $w(z)$ is a complex function, and equations (1) and (2) can be written as follows:

$$\frac{\mathrm{d}^2 w}{\mathrm{d}z^2}+q\alpha^2k^2w+q\frac{\mathrm{d}}{\mathrm{d}z}\left(\varepsilon_2\frac{\mathrm{d}w}{\mathrm{d}z}\right)+q\alpha^2k^2\varepsilon_1 w=0 \tag{6}$$

$$\tau(z)=E_{01}\left(\frac{1}{q}\frac{\mathrm{d}w}{\mathrm{d}z}+\varepsilon_2\frac{\mathrm{d}w}{\mathrm{d}z}\right) \tag{7}$$

where $q=1/(1+\mathrm{i}\kappa)$, $\alpha=C_2/C_{01}$, $k=\omega/C_2$, $\kappa=\xi\omega/E_{01}$, $C_{01}=\sqrt{E_{01}/\rho_{01}}$, $C_2=\sqrt{E_2/\rho_2}$.

The modulus of elasticity E_2 and the density ρ_2 introduced in equations (6) and (7) will be explained in Section 3 [see Figure 1 and equations (28) and (29)].
Introducing new variables

$$w=y_1$$
$$\frac{dw}{dz}=y_2 \tag{8}$$

the second order differential equation (6) can be reduced to the following first order matrix differential equation:

$$\frac{dy}{dz}=A\mathbf{y}+\mathbf{g}(\mathbf{y}(z)) \tag{9}$$

where

$$A=\begin{bmatrix} 0 & 1 \\ -q\alpha^2k^2 & 0 \end{bmatrix}, \mathbf{y}=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \mathbf{g}(\mathbf{y})=\begin{bmatrix} 0 \\ g_2(\mathbf{y}) \end{bmatrix} g_2(\mathbf{y})=-q\frac{d}{dz}(\varepsilon_2 y_2)-q\alpha^2k^2\varepsilon_1 y_1 \tag{10}$$

Using the fundamental matrix[8] $\Phi(z)$, which satisfies the condition

$$\Phi(0)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

one can write equation (9) in the form

$$\mathbf{y}(z)=\Phi(z)\mathbf{y}_0+\int_0^z \Phi(z-\zeta)\mathbf{g}(\mathbf{y}(\zeta))d\zeta \tag{12}$$

where

$$\mathbf{y}(0)=y_0\equiv\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, \Phi(z)=\begin{bmatrix} \dfrac{dG}{dz} & G \\ -q\alpha^2k^2G & \dfrac{dG}{dz} \end{bmatrix} \tag{13a,b}$$

$$G(z)=\frac{\sinh(\sqrt{-q}\alpha kZ)}{\sqrt{-q}\alpha k} \tag{13c}$$

By virtue of integration by parts in equation (12) one obtains

$$w=\frac{dG}{dz}Y_1+(1+q\varepsilon_2(0))GY_2-q\int_0^z\left(\frac{\partial G}{\partial z}\varepsilon_2\frac{dw}{d\zeta}+\alpha^2k^2G\varepsilon_1 w\right)d\zeta \tag{14a}$$

$$\frac{dw}{dz}=-q\alpha^2k^2GY_1+(1+q\varepsilon_2(0))\frac{dG}{dz}Y_2-q\varepsilon_2\frac{dw}{dz}+q\alpha^2k^2\int_0^z\left(qG\varepsilon_2\frac{dw}{d\zeta}-\frac{\partial G}{\partial z}\varepsilon_1 w\right)d\zeta \tag{14b}$$

Statistical averaging of equation (6) yields

$$\frac{d^2\langle w\rangle}{dz^2}+q\alpha^2k^2\langle w\rangle+q\frac{d}{dz}\left\langle\varepsilon_2\frac{dw}{dz}\right\rangle+q\alpha^2k^2\langle\varepsilon_1 w\rangle=0 \tag{15}$$

In obtaining the equations for the averaged displacement and stress the following approximation is adopted:

$$\langle\varepsilon_i(z)\varepsilon_j(\zeta)w(z)\rangle\simeq K_{ij}(z-\zeta)\langle w(z)\rangle \quad (i,j=1,2) \tag{16}$$

where $K_{ij}(z-\zeta)=\langle \varepsilon_i(z)\varepsilon_j(\zeta)\rangle$ and $\varepsilon_j(z)$ are stationary random functions. This approximation is equivalent to the 'local independence' assumption of Bourret commonly employed in the literature.[10-12]

By equations (14), (16) and (15) one can obtain the following integro-differential equation for the average displacement:

$$(1-q^2K_{22}(0))\frac{d^2\langle w\rangle}{dz^2}+q\alpha^2k^2\langle w\rangle+B_1(z)\,Y_1+B_2(z)\,Y_2+$$

$$q^2\alpha^2k^2\int_0^z L(z-\zeta)\langle w(\zeta)\rangle\,d\zeta=0 \tag{17}$$

where

$$B_1(z)=-q^2\alpha^2k^2K_{12}(z)\frac{dG}{dz} \tag{18a}$$

$$B_2(z)=-q^2\frac{d}{dz}\left(K_{22}(z)\frac{dG}{dz}\right)+q^2\alpha^2k^2K_{12}(z)G(z)+q^3\alpha^2k^2K_{22}(z)G(z) \tag{18b}$$

$$L(z-\zeta)=qK_{22}(z-\zeta)G(z-\zeta)\frac{d^2}{d\zeta^2}-2K_{12}(z-\zeta)\frac{\partial G}{\partial z}\frac{d}{d\zeta}-\alpha^2k^2K_{11}(z-\zeta)G(z-\zeta) \tag{18c}$$

The correlation function which has been used in a number of investigations and fits experimental data the best[1,2,11,13] is the exponential function

$$K_{ij}(z-\zeta)=\sigma_{ij}\exp(-\beta|z-\zeta|)\quad (i,j=1,2) \tag{19}$$

where $\sigma_{ij}=K_{ij}(0)$ denotes variance and $1/\beta>0$ is the radius of correlation.

3. GENERAL SOLUTION

The solution of equation (17), by use of the Laplace transform[9]

$$f^*(p)=\int_0^\infty \langle f(z)\rangle e^{-pz}\,dz \tag{20}$$

can be written in the form

$$W^*(\tilde{p})=\frac{U_3(\tilde{p})}{U_4(\tilde{p})}\,Y_1+\frac{U_2(\tilde{p})}{kU_4(\tilde{p})}\,Y_2 \tag{21}$$

where

$$U_2(\tilde{p})=(1-q^2\sigma_{22})\tilde{p}^2+\tilde{\beta}(2-q^2\sigma_{22})\tilde{p}+\tilde{\beta}^2+q\alpha^2(1-q\sigma_{12}) \tag{22a}$$

$$U_3(\tilde{p})=(1-q^2\sigma_{22})\tilde{p}^3+2\tilde{\beta}(1-q^2\sigma_{22})\tilde{p}^2+[\tilde{\beta}^2(1-q^2\sigma_{22})+q\alpha^2(1-q\sigma_{12})]\tilde{p}-\tilde{\beta}q^2\alpha^2\sigma_{12} \tag{22b}$$

$$U_4(\tilde{p})=(1-q^2\sigma_{22})\tilde{p}^4+2\tilde{\beta}(1-q^2\sigma_{22})\tilde{p}^3+[\tilde{\beta}(1-q^2\sigma_{22})+2q\alpha^2(1-q\sigma_{12})]\tilde{p}^2$$
$$+2q\tilde{\beta}\alpha^2(1-q\sigma_{12})\tilde{p}+\tilde{\beta}q\alpha^2+q^2\alpha^4(1-\sigma_{11}) \tag{22c}$$

All parameters in equations (22) are non-dimensional and $\tilde{\beta}=\beta/k$, $\tilde{p}=p/k$.

By the residue theorem and Jordan's lemma[14] one obtains the inverse Laplace transform of equation (21) as

$$\langle w(z)\rangle = Y_1f_1(z)+\frac{1}{k}\,Y_2f_2(z) \tag{23}$$

where

$$f_1(z)=\sum_{n=1}^{4}\frac{U_3(\tilde{p}_n)}{U_4'(\tilde{p}_n)}\exp(\tilde{p}_n kz) \tag{24a}$$

$$f_2(z)=\sum_{n=1}^{4}\frac{U_2(\tilde{p}_n)}{U_4'(\tilde{p}_n)}\exp(\tilde{p}_n kz) \tag{24b}$$

A prime in this paper denotes the derivative and in equations (24) we use the notation

$$U_4'(\tilde{p}_n)=\left.\frac{\mathrm{d}U_4(\tilde{p})}{\mathrm{d}\tilde{p}}\right|_{\tilde{p}=\tilde{p}_n} \tag{25}$$

$\tilde{p}_n$ $(n=1,\ldots,4)$ are the roots of the fourth degree polynomial equation with complex coefficients

$$U_4(\tilde{p})=0 \tag{26}$$

Owing to the construction of the solution (23) a check on the calculation is possible at this stage. It should be

$$f_1(0)=1,\ f_2(0)=0$$
$$f_1'(0)=0,\ f_2'(0)=1 \tag{27}$$

Now we assume that the solution (23) is valid only for $0\leqslant z\leqslant h$ and the solution for the half-space $z\geqslant h$ will be obtained from the deterministic equations

$$\frac{\mathrm{d}^2w}{\mathrm{d}z^2}+qk^2w=0 \tag{28}$$

$$\tau(z)=\frac{E_2}{q}\frac{\mathrm{d}w}{\mathrm{d}z} \tag{29}$$

Using the radiation condition the solution of equation (28) takes the form

$$w=Y_3\exp(-\sqrt{-q}kz) \tag{30}$$

where the square root, $\sqrt{-q}$, has the positive real value

$$\sqrt{-q}=\frac{1}{\sqrt{2(1+\kappa^2)}}\left((-1+\sqrt{1+\kappa^2})^{\frac{1}{2}}+i(1+\sqrt{1-\kappa^2})^{\frac{1}{2}}\right) \tag{31}$$

Averaging of equation (7) yields

$$\langle\tau(z)\rangle=E_{01}\left(\frac{1}{q}\frac{\mathrm{d}\langle w\rangle}{\mathrm{d}z}+\left\langle\varepsilon_2\frac{\mathrm{d}w}{\mathrm{d}z}\right\rangle\right)\quad\text{for}\quad 0\leqslant z\leqslant h. \tag{32}$$

Substituting equations (14b) and (23) into equation (32) yields

$$\frac{\langle\tau(z)\rangle}{E_{01}}=g_1(z)Y_1+g_2(z)Y_2\quad\text{for}\quad 0\leqslant z\leqslant h \tag{33}$$

where

$$g_1(z)=\frac{1}{q}(1-q^2K_{22}(0))f_1'(z)+q\alpha^2k_0^2\int_0^z\left(qGK_{22}f_1'(\zeta)-\frac{\partial G}{\partial z}K_{12}f_1(\zeta)\right)\mathrm{d}\zeta$$

$$g_2(z)=\frac{1}{q}(1-q^2K_{22}(0))f_2'(z)+qK_{22}\frac{\mathrm{d}G}{\mathrm{d}z}+q\alpha^2k\int_0^z\left(qGK_{22}f_2'(\zeta)-\frac{\partial G}{\partial z}K_{12}f_2(\zeta)\right)\mathrm{d}\zeta \tag{34}$$

Equations (29) and (30) yield

$$\frac{\tau(z)}{E_{01}} = Y_3 \frac{E_2}{E_{01}} \frac{-k}{\sqrt{-q}} \exp(-\sqrt{-q}kz) \quad \text{for} \quad z \geqslant h \tag{35}$$

4. SOLUTION OF THE PROBLEM

From the boundary condition (4) and equation (33) one obtains

$$Y_2 = q \frac{\tau_0}{E_{01}} \tag{36}$$

Continuity conditions

$$\langle w(h^-) \rangle = w(h^+) \tag{37}$$

$$\langle \tau(h^-) \rangle = \tau(h^+) \tag{38}$$

and equations (23), (30), (33), (35) and (36) yield, after some straightforward but lengthy operations, the

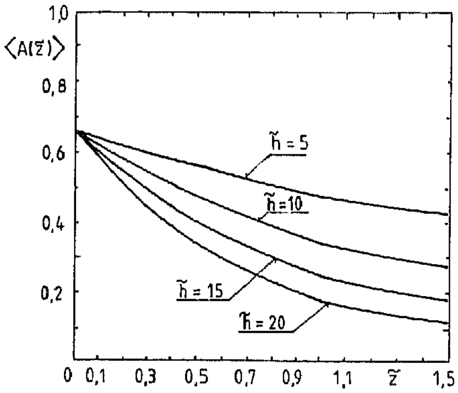

Figure 2. Distribution of the mean amplitude in the half-space for $\kappa = 0{\cdot}09$, $\alpha = \tilde{E} = 1{\cdot}5$, $\sigma_{22} = \sigma_{11} = \sigma_{12} = \sigma = 0$

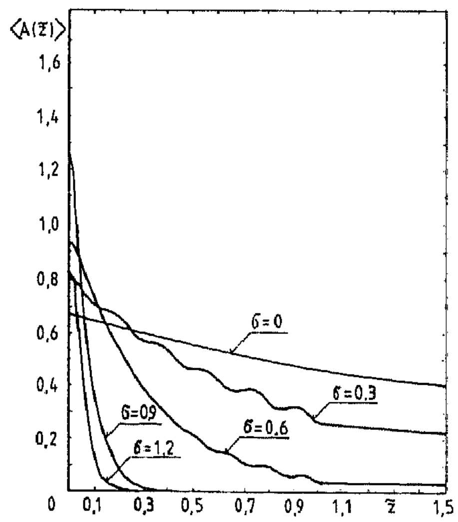

Figure 3. Distribution of the mean amplitude in the half-space, for $\bar{\beta} = 5$, $\kappa = 0{\cdot}05$, $\alpha = \tilde{E} = 1{\cdot}5$, $\bar{h} = 10$, $\sigma_{22} = \sigma_{11} = \sigma$, $\sigma_{12} = 0$

solution for the average displacements in dimensionless form

$$\frac{E_{01}}{\tau_0}k\langle w(\tilde{z})\rangle=\begin{cases} F_0\phi_1(\tilde{z})+q\phi_2(\tilde{z}), & 0\leqslant\tilde{z}\leqslant 1 \\ (F_0\phi_1(1)+q\phi_2(1))\exp(-\sqrt{-q}\tilde{h}(\tilde{z}-1)), & \tilde{z}\geqslant 1 \end{cases} \tag{39}$$

where

$$\tilde{z}=\frac{z}{h} \tag{40a}$$

$$\phi_1(\tilde{z})=\sum_{n=1}^{4}\frac{U_3(\tilde{p}_n)}{U'_4(\tilde{p}_n)}\exp(\tilde{p}_n\tilde{h}\tilde{z}) \tag{40b}$$

$$\phi_2(\tilde{z})=\sum_{n=1}^{4}\frac{U_2(\tilde{p}_n)}{U'_4(\tilde{p}_n)}\exp(\tilde{p}_n\tilde{h}\tilde{z}) \tag{40c}$$

$$\phi_i(1)=\phi_i(\tilde{z})|_{\tilde{z}=1}\quad (i=1,2) \tag{40(d,e)}$$

$$F_0=-\frac{\tilde{E}\sqrt{-q}\phi_2(1)+(1-q^2\sigma_{22})\phi'_2(1)+q^2\alpha^2\Psi_2+q^2N_0}{\tilde{E}\frac{\sqrt{-q}}{q}\phi_1(1)+\frac{1}{q}(1-q^2\sigma_{22})\phi'_1(1)+q\alpha^2\Psi_1} \tag{40(f)}$$

$$\Psi_1=\sum_{n=1}^{4}\frac{U_3(\tilde{p}_n)}{U'_4(\tilde{p}_n)}(S_{1n}-S_{2n}) \tag{40(g)}$$

$$\Psi_2=\sum_{n=1}^{4}\frac{U_2(\tilde{p}_n)}{U'_4(\tilde{p}_n)}(S_{1n}-S_{2n}) \tag{40(h)}$$

$$S_{1n}=\frac{1}{2}\left(\frac{1}{\alpha}\frac{q}{\sqrt{-q}}\sigma_{22}\tilde{p}_n-\sigma_{12}\right)\frac{\exp(\tilde{p}_n\tilde{h})-\exp((-\tilde{\beta}+\alpha\sqrt{-q})\tilde{h})}{\tilde{p}_n+\tilde{\beta}-\alpha\sqrt{-q}} \tag{40(i)}$$

$$S_{2n}=\frac{1}{2}\left(\frac{1}{\alpha}\frac{q}{\sqrt{-q}}\sigma_{22}\tilde{p}_n+\sigma_{12}\right)\frac{\exp(\tilde{p}_n\tilde{h})-\exp((-\tilde{\beta}-\alpha\sqrt{-q})\tilde{h})}{\tilde{p}_n+\tilde{\beta}+\alpha\sqrt{-q}} \tag{40(j)}$$

$$\phi'_1(1)=\sum_{n=1}^{4}\tilde{p}_n\frac{U_3(\tilde{p}_n)}{U'_4(\tilde{p}_n)}\exp(\tilde{p}_n\tilde{h}) \tag{40(k)}$$

$$\phi'_2(1)=\sum_{n=1}^{4}\tilde{p}_n\frac{U_2(\tilde{p}_n)}{U'_4(\tilde{p}_n)}\exp(\tilde{p}_n\tilde{h}) \tag{40(l)}$$

$$N_0=\frac{\sigma_{22}}{2}(\exp((-\tilde{\beta}+\alpha\sqrt{-q})\tilde{h})+\exp((-\tilde{\beta}-\alpha\sqrt{-q})\tilde{h})) \tag{40(m)}$$

$$\tilde{E}=\frac{E_2}{E_{01}} \tag{40n}$$

5. NUMERICAL RESULTS AND DISCUSSION

The harmonically loaded viscoelastic medium, which is considered in this paper, is a randomly non-homogeneous layer over-lying a homogeneous half-space.

In the paper we assume that the modulus of elasticity $E(z,\gamma)$ and the density $\rho(z,\gamma)$ of the layer are stationary random functions, which are approximately described by the mean values E_{01} and ρ_{01} and correlation functions $K_{ij}(z-\zeta)$, $(i,j=1,2)$. In obtaining the equations for the averaged displacement and stress the Bourret approximation [equation (16)] is adopted. To obtain numerical results the exponential correlation functions have been used [equation (19)]. Equation (39) provides the solution for average displacements through the non-dimensional parameters: σ_{22}, σ_{11}, σ_{12}, $\tilde{\beta}=\beta/k$, κ, $\alpha=C_2/C_0$, $\tilde{E}=E_2/E_{01}$, $\tilde{h}=kh$ and $\tilde{z}=z/h$. The solution is expressed in terms of the complex number $q=(1-i\kappa)/(1+\kappa^2)$ and the

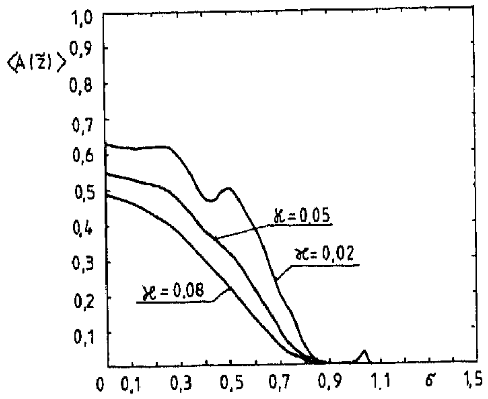

Figure 4. Effect of variance on the mean amplitude for $\tilde{z}=0{\cdot}5$, $\tilde{\beta}=5$, $\alpha=\tilde{E}=1{\cdot}5$, $\tilde{h}=10$, $\sigma_{22}=\sigma_{11}=\sigma$, $\sigma_{12}=0$

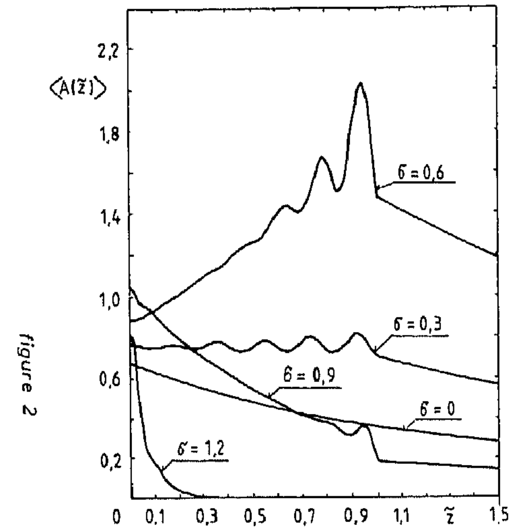

Figure 5. Distribution of the mean amplitude in the half-space for $\tilde{\beta}=5$, $\kappa=0{\cdot}09$, $\alpha=\tilde{E}=1{\cdot}5$, $\tilde{h}=10$, $\sigma_{22}=\sigma_{11}=\sigma_{12}=\sigma$

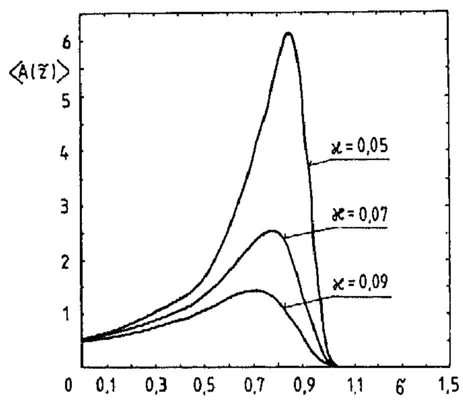

Figure 6. Effect of variance on the mean amplitude for $\tilde{z}=0{\cdot}5$, $\tilde{\beta}=5$, $\alpha=\tilde{E}=1{\cdot}5$, $\tilde{h}=10$, $\sigma_{22}=\sigma_{11}=\sigma_{12}=\sigma$

complex roots of equation (26) $\tilde{p}_n = p_n/k$ ($n=1, 2, 3, 4$). It is to be noted that the IBM Manual of Scientific Packages has programs for finding the roots of polynomials with complex coefficients.[15] Numerical results which demonstrate the effect of the randomness of medium parameters on the mean wave propagation are presented here in terms of the non-dimensional mean displacement amplitude

$$\langle A(\tilde{z})\rangle = \frac{E_{01}}{\tau_0} k(\langle w(\tilde{z})\rangle \langle w(\tilde{z})\rangle^*)^{\frac{1}{2}} \tag{41}$$

where the star denotes the complex conjugate number. As expected, the mean wave in the stochastic case, $\sigma_{ij} \neq 0$, is damped more rapidly with depth $\tilde{z}$ than the case of the deterministic medium, $\sigma_{ij}=0$ (see Figures 2 and 3). In Figures 3 and 4 it is observed that the effect of the inhomogeneities gets stronger as the variance increases. However, this conclusion is true only when the non-dimensional material parameters $\varepsilon_2(z, \gamma)$ and $\varepsilon_1(z, \gamma)$ are uncorrelated, $\sigma_{12}=0$. If we admit parameters ε_2 and ε_1 whose fluctuations are correlated, $\sigma_{12} \neq 0$, the effective behaviour of the mean wave can differ considerably from the above results (see Figures 3 and 5). From a study of Figures 6, 7 and 5 it is seen that the case of correlated fluctuations of parameters gives rise to amplification of waves for some region of variance σ, and a high peak for some value of $\sigma \ll 1$. If the variances σ_{22} and σ_{11} are greater than one, $\sigma > 1$, the solutions for the mean displacements have similar peculiarities for the case of $\sigma_{12} \neq 0$ as well as for $\sigma_{12}=0$. The mean wave is damped with depth very strongly for $\sigma_{ij} > 1$ (see Figures 3, 4, 5 and 6). Figure 8 shows the effect of correlation length, $1/\beta$, on the mean displacement amplitude in the middle of the layer, $\tilde{z}=0{\cdot}5$. It can be seen that a high peak exists for some value of $\tilde{\beta}$.

On the contrary, as in Figure 2, Figure 9 shows an increase of the amplification waves with increasing non-dimensional frequency $\tilde{h}=kh$. An important numerical result of this paper is that, in the case of correlated

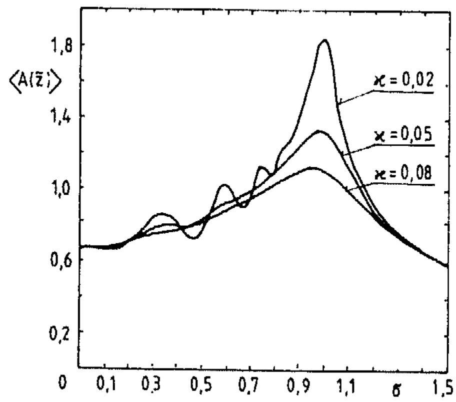

Figure 7. Effect of variance on the mean amplitude for $\tilde{z}=0$, $\tilde{\beta}=5$, $\alpha=\tilde{E}=1{\cdot}5$, $\tilde{h}=10$, $\sigma_{22}=\sigma_{11}=\sigma_{12}=\sigma$

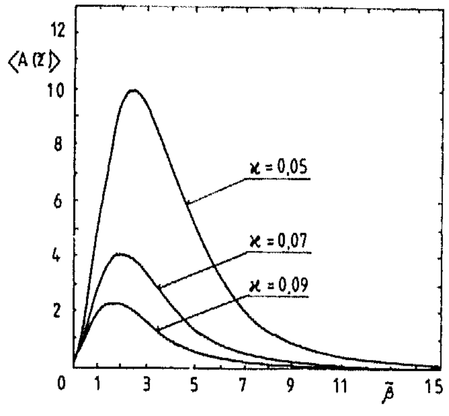

Figure 8. Effect of correlation length on the mean amplitude for $\tilde{z}=0{\cdot}5$, $\alpha=\tilde{E}=1{\cdot}5$, $\tilde{h}=10$, $\sigma_{22}=\sigma_{11}=\sigma_{12}=\sigma=0{\cdot}9$

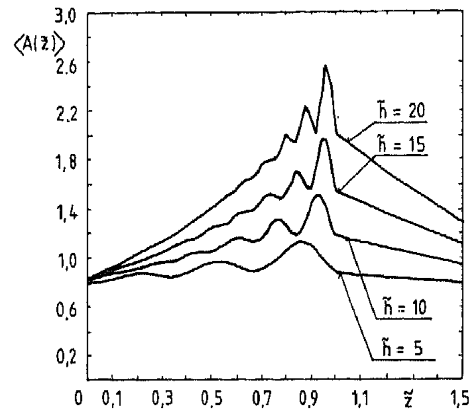

Figure 9. Distribution of the mean amplitude in the half-space for $\bar{\beta}=5$, $\kappa=0{\cdot}09$, $\alpha=\tilde{E}=1{\cdot}5$, $\sigma_{22}=\sigma_{11}=\sigma_{12}=\sigma=0{\cdot}5$

fluctuations of parameters of the random medium, the modulus of elasticity and the density, one obtains wave amplification for some region of variance. The method presented here provides a tool for accounting quantitatively for the mean properties of wave damping in a viscoelastic semi-infinite random medium.

REFERENCES

1. T. T. Soong, *Random Differential Equations in Science and Engineering*, Academic Press, New York, 1973.
2. U. Frish, 'Wave propagation in random media', in *Probabilistic Methods in Applied Mathematics*, Vol 1 (Ed. A. T. Bharucha-Reid), Academic Press, New York, 1968.
3. G. Gazetas, 'Static and dynamic displacements of foundations on heterogeneous multilayered soils', *Géotechnique* **30**, 159–177 (1980).
4. C. C. Spyrakos and D. E. Beskos, 'Dynamic response of rigid strip foundations by a time domain boundary element method', *Int. j. numer. methods eng.* **23**, 1547–1565 (1986).
5. A. Ben-Menahem and S. J. Singh, *Seismic Waves and Sources*, Springer-Verlag, Berlin, 1981.
6. J. T. Christian, 'Probabilistic soil dynamics: state of the art', *J. geotech. eng. div. ASCE* **106**, 386–397 (1980).
7. J. L. Nowinski, 'On the non-local aspects of stress in a viscoelastic medium', *Int. j. non-linear mech.* **21**, 439–446 (1986).
8. F. C. Karal and J. B. Keller, 'Elastic, electromagnetic and other waves in a random medium', *J. math. phys.* **5**, 537–547 (1964).
9. W. E. Boyce and R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, Wiley, New York, 1977.
10. R. C. Bourret, 'Stochastically perturbed fields with applications to wave propagation in random media', *Nuovo cimento* **26**, 1–31 (1962).
11. J. M. Richardson, 'The application of truncated hierarchy techniques in the solution of a stochastic linear differential equation', in *Proc symp. appl. math.* Providence, R.I. **16**, 290–302 (1964).
12. V. I. Tatarski, *Wave Propagation in a Turbulent Medium*, McGraw-Hill, New York, 1961.
13. Z. Hryniewicz and A. J. Hermans, 'Effective solution for Rayleigh-type waves in a randomly inhomogeneous medium', *J. sound vib* **115**, 351–362 (1987).
14. H. F. Weinberger, *A First Course in Partial Differential Equations*, Wiley, New York, 1965.
15. J. A. Grant and G. D. Hitchins, 'Two algorithms for the solution of polynomial equations to limiting machine-precision', *Comput. j.* **18**, 258–264 (1975).